Programming Languages

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Math Example

$$2 + 3 * 7$$

Math Example

$$2 + 3 * 7$$

$$2 + 21$$

$$23$$

Multiplication

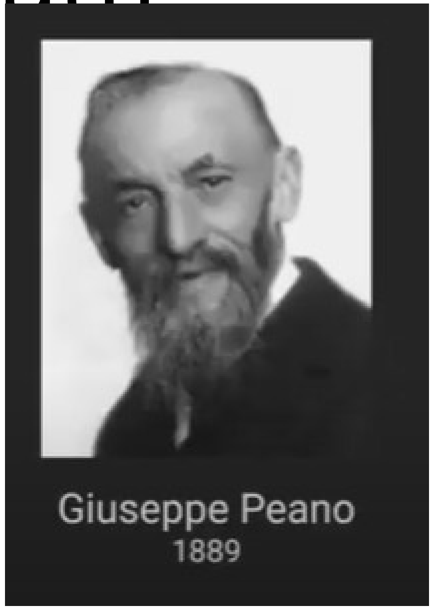
- $2 + 3 * 7$
- $2 + 7 + 7 + 7$
- Multiplication is defined as multiple addition
- Some rules are defined in terms of other rules
    - *Multiplication is redundant*

Rules

- What is the minimum subset of rules necessary ?
  - We can call this minimum subset *axioms*
    - Greek axioma “that which is self evident”
  - We can call derived rules *theorems*
    - Greek theorema “a preposition to be proved”

Question

- What are axioms for math?
  - The minimum (non – redundant) set of rules to define all of math



Peano Axioms

- 0 is a ***natural number***
- $x = x$
- If $x = y$ then $y = x$
- If $x = y$ and $y = z$ then $x = z$
- If b is a natural number and $a = b$ then a is also a natural number
- There's a function S, such that S(n) is a natural number
- $m = n$ if and if $S(m) = S(n)$
- There's no n such that $S(n) = 0$
- If k is a set such that
  - 0 is in k
  - If n is in K means that $S(n)$ is in K
  - Then K contains every natural number

Peano Numbers (syntactic sugar)

- 0
- 1 := S(0)
- 2 := S(1) := S(S(0))
- 3 : = S(S(S(0)))

Syntactic sugar

- Convenience rules / symbols that don’t need to be reduced to their most primitive form

Theorem of Addition

- Addition can be thought of as an operation that maps two natural numbers to another natural number

- Syntax = a + b

① $a + 0 = a$

② $a + S(b) = S(a + b)$

Addition Example

- 3 + 2
- S(S(S(0))) + S(S(0))
- S(S(S(S(0))) + S(0))
- S(S(S(S(S(0))) + 0))
- S(S(S(S(S(0)))))
- 5

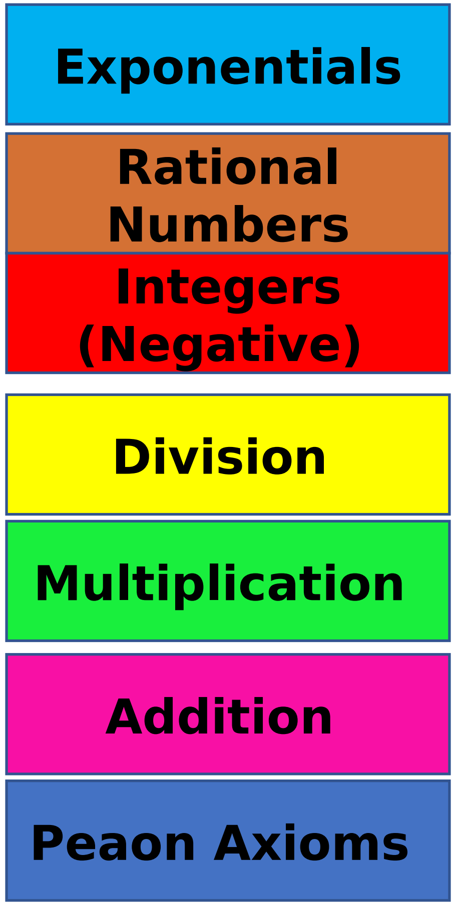
Theorem of Multiplication

- Multiplication can also be thought of as an operation that maps two natural numbers to another natural number
- Syntax a * b

① $a * 0 = 0$

② $a * S(b) = a + (a * b)$

Axiom Towers



Symbols

- It might be tempting to think of symbols as separate from axioms / theorems
- In reality symbols don't mean anything without the rules and the rules only make sense in terms of symbols

0 x 20

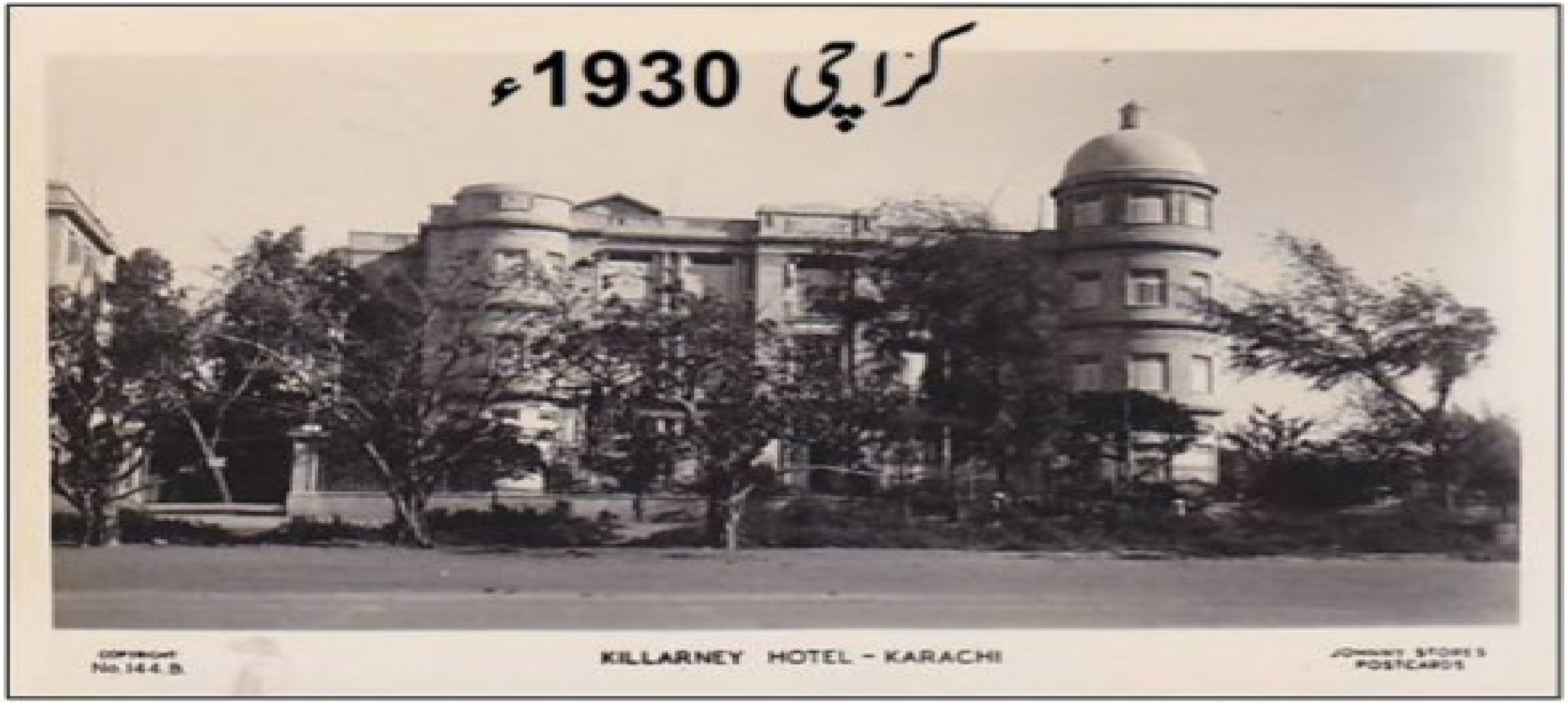
Math is discovered, not invented

- Math is the discovery of upper levels in axiom towers that are obscured by clouds
- Technically axioms are invented in that they are arbitrary but inventing axioms isn’t what we think about when we think about math.

Recap

- Axioms are “self Evident” (taken as given) rules
- Theorems are derived (redundant rules)
- Axioms and Theorems stack up to build axiom towers
- Some symbols are syntactic sugar
- Symbols and rules are intrinsically related
- Math is the discovery of the consequences of foundational axioms
- Axioms are arbitrary but some axiom towers are more useful than others

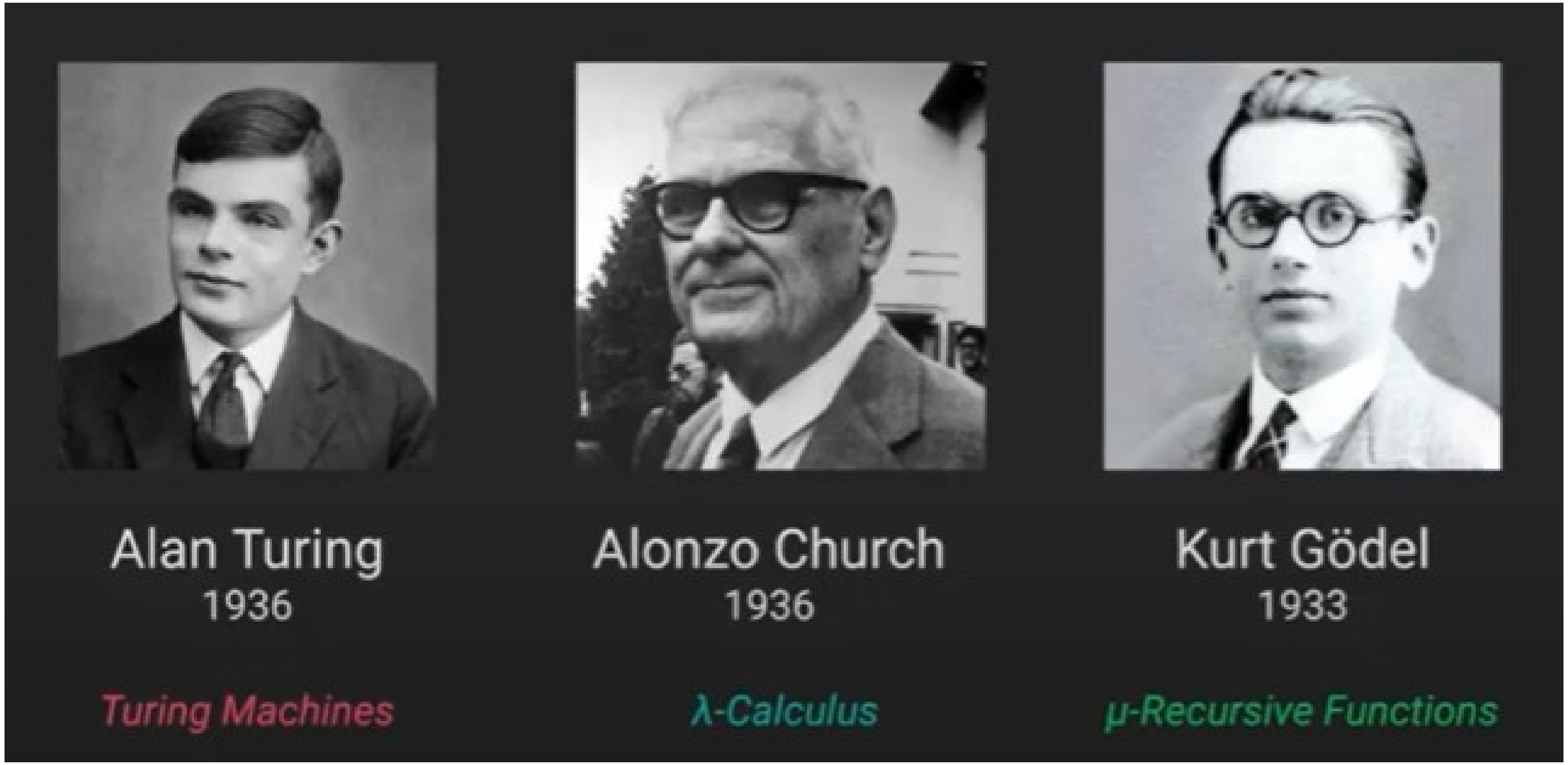
Thinking About Computation



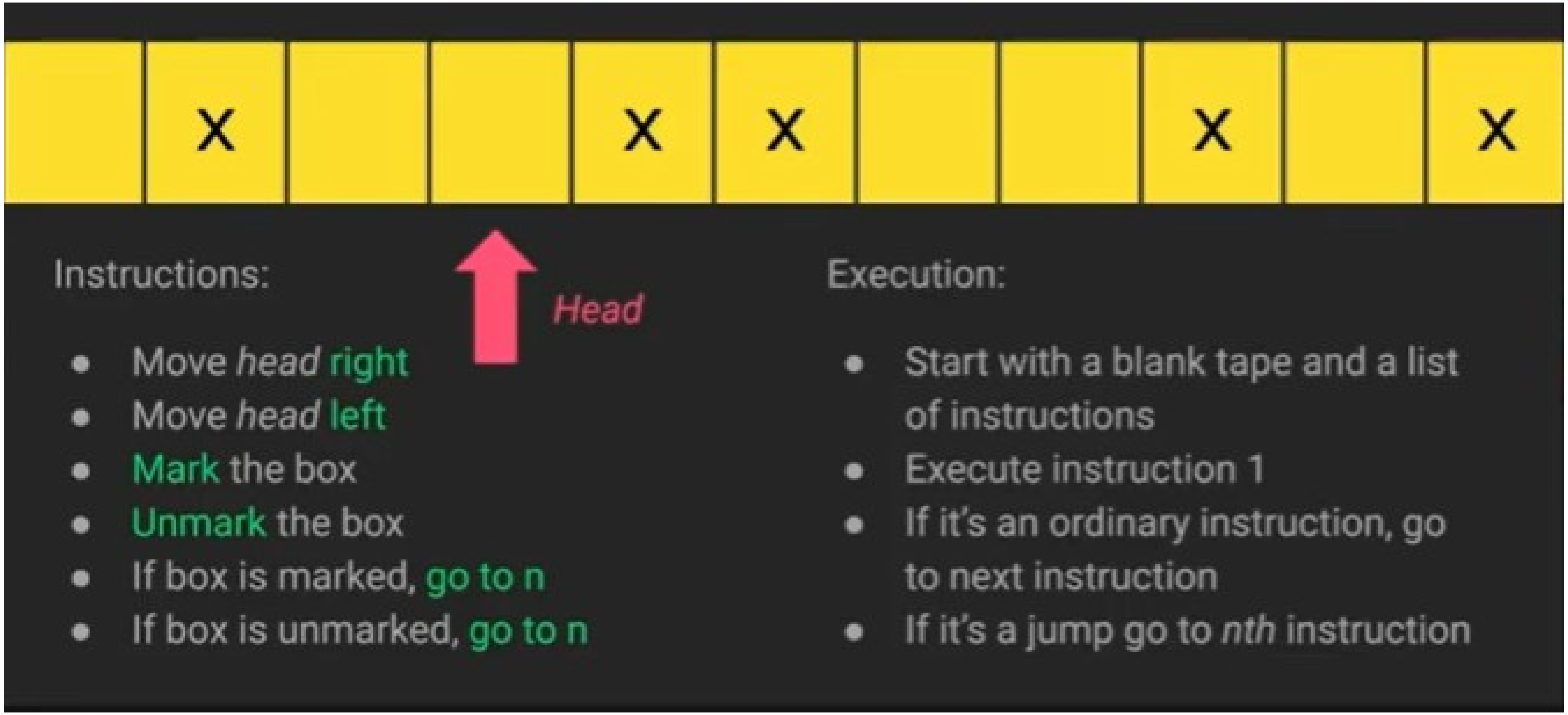
Algorithms already existed in 1930’s

- 2000 BC – Egyptians: algorithms for multiplying two numbers
- 1600 BC – Babylonians: factorization and finding square roots
- 300 BC – Euclid's algorithm (greatest common factor)
- 200 BC – the Sieve of Eratosthenes (prime numbers)
- 820 AD – Al-Khawarizmi: solving linear equations and quadratic equations (the word algorithm comes from his name)

Exactly at the same Time



Turing Machines



Turing Created an Axiom Tower for Computing

- An algorithm is “computable” if and only if it can be encoded as a Turing Machine
- Turing Showed this before the existence of electrical computers
- He did this when he was 24 years old

Some Observations

1) You need an infinite tape and a program.
2) You are constantly modifying the tape (state)
3) The tape/state determines how the program runs(jumps)
    1) The behaviour of the program is changed with every tape modification
    2) Reasoning about the behaviour of the program requires understanding of the state of the tape at every moment of modification.

Turing Completeness

- You can imagine other axiom towers(e.g. different set of instructions for our Turing machine)
- If the axiom tower can simulate a Turing machine, it is describe Turing complete and therefore can compute anything that is computable.